**1.1 Using Differences to Identify Patterns**

A **number sequence** is a string of numbers, or terms, in a certain order. Often there will be three dots (…) after the last given number. The three dots (…) indicate that there are more terms not listed.

If the difference from one term to the next in a number sequence is always the same, the difference is called the **constant difference**.

***Example 2a: Number Sequence***

| | **1** | | **4** | | **9** | | **16** | | **25** | **…** |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 4 minus 1 | | 9 minus 4 | | 16 minus 9 | | 25 minus 16 | | |
| First Difference | | =+3 | | =+5 | | =+7 | | =+9 | | Not Constant |
| | | | 5 minus 3 | | 7 minus 5 | | 9 minus 7 | | | |
| Second Difference | | | =+2 | | =+2 | | =+2 | | | Constant Difference |

A **Table** of the sequence in Example 2a.

| **Terms** | | **First Difference, 1st** | | **Second Difference, 2nd** | |
|---|---|---|---|---|---|
| 1 | | | | | |
| | | 4 minus 1 = +3 | 3 | | |
| 4 | | | | 5 minus 3 = +2 | 2 |
| | | 9 minus 4 = +5 | 5 | | The constant difference is 2. |
| 9 | | | | 7 minus 5 = +2 | 2 |
| | | 16 minus 9 = +7 | 7 | | |
| 16 | | | | 9 minus 7 = +2 | 2 |
| | | 25 minus 16 = +9 | 9 | | |
| 25 | | | | | |
| | | | | | |
| … | | | | | |

Notice the sequence increases from left to right with a positive constant difference.

Challenge: What would happen if the constant difference was negative?

A **conjecture** is a statement about observations that is believed to be true.

When mathematicians make a conjecture, they try to either prove that the conjecture is true or find a counterexample to show that the conjecture is not true.

You may make conjectures when studying a pattern such as a number sequence.

Problem solving strategies include solving a simpler problem, making a table or chart, and looking for a pattern.

**1.2 Variables, Expressions and Equation**

**Variables** are letters that are used to represent quantities in algebra. For example, the letter *h* can be used to represent the number of hours a bike will be rented.

The cost of renting a bike can be written as an algebraic expression.

$8 + 3(h)$ Algebraic Expression for the Cost of Renting a Bike

An **algebraic expression** is formed by combining numbers and variables using the operations of addition, subtraction, multiplication, division, powers and roots.

The **value of the** algebraic expression can be computed when the variable *h* is known. The cost of renting a bike for 5 hours is 8 + 3(5) = 8 + 15 = 23. The cost of renting a bike is $23.

**Cost of Renting a Bike by the Hour from 1hr to 5hrs**

**8 + 3*h*** ***h*** is a variable that represents the number of rental hours

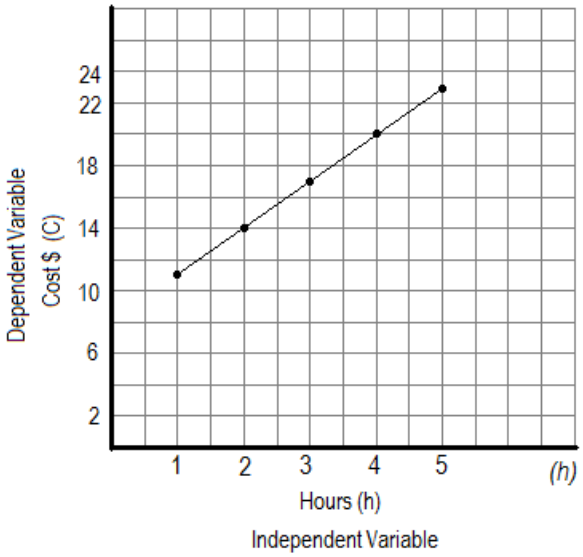
| Number of Hours (hrs) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Cost (C) $ | 11 | 14 | 17 | 20 | 23 |

How many ways to represent 3 times h:

$3h$ $3(h)$ $3 \bullet h$ $3 \times h$

8 + 3(*h*), h = 1, 2, 3, 4, 5 and 6.
8 + 3(*1*) = 11
8 + 3(*2*) = 14
8 + 3(*3*) = 17
8 + 3(*4*) = 20
8 + 3(*5*) = 23

- Value of the expression
- Table or Chart
- Plot of table value




When you connect the points plotted above, the graph of the equation is Cost=8+3h is a straight line. For this reason, the equation is called a **linear equation**.

In the equation C=8+3h, h is called the **independent variable**, and C is the **dependent variable**. The value of C depends on the value of h.

**Challenge**

- How much will the rental cost for 1.5hrs?
- If we extended the line to the y-axis (Cost $), what is the significance of that value?
- Can we have negative hours?

Two algebraic expressions separated by an equal sign '=' form an **equation**. When two variables represent the same quantity, an equation can be written.

$8 + 3h = 38$ **Algebraic Equation**

**Guess-and-check strategy**

Try 9, substitute *h* for 9 in the expression 8 + 3h: 8 + 3(9) = 8 + 27 = 35, 35≠38

Try 10, substitute *h* for 10 in the expression 8 + 3h: 8 + 3(10) = 8 + 30 = 38, 38=38, therefore the solution to the algebraic equation is h=10.

The **solutions** to an equation are all values of the variables that give a true equation when substituted for the variables. In many equations, such as the equation above there is only one solution.

**1.3 The Algebraic Order of Operations**

1. Perform all operations within grouping symbols (), {}, [], from innermost to outward.
2. Perform all operations with exponents.
3. Perform all multiplications and divisions in order from left to right.
4. Perform all the additions and subtractions in order from left to right.

Parentheses '**()**', brackets '**[]**', braces '**{}**' and the '**–**' fraction bar are called **grouping symbols** or inclusion symbols. Treat any grouped number or variables as a single quantity. Operations should always be done within the innermost grouping symbols first. Then work outward.

Examples

4[(3+**2×3**)-5]+7

$$\frac{4\left[\left(3+2\times 3\right)-5\right]+7}{2\times 6^2}$$

4[(<u>3+**6**</u>)-5]+7

$$\frac{2_3}{2\times 6^2}$$

4[(9)-5]+7

16+7

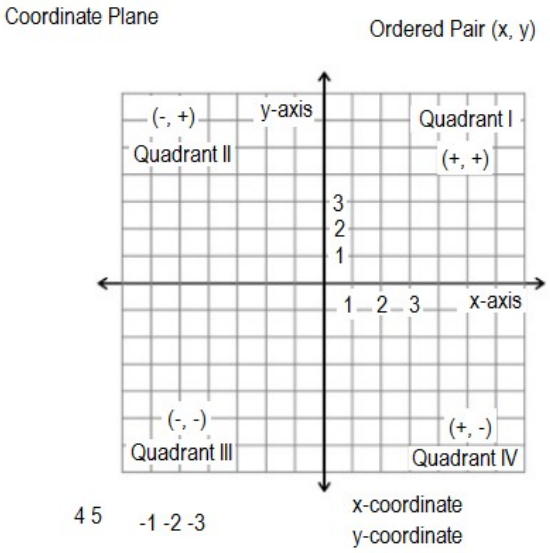
23

$$\frac{2_3}{2 \times 6 \times 6}$$

$$\frac{2_3}{2 \times 36}$$

$$\frac{2_3}{72} \quad \text{or} \quad 0.32$$

**1.4 Graphing with Coordinates**




**1.5 Representing Linear Patterns**

Cost of Renting a Bike

**8 + 3*h*** ***h*** is a variable that represents the number of rental hours

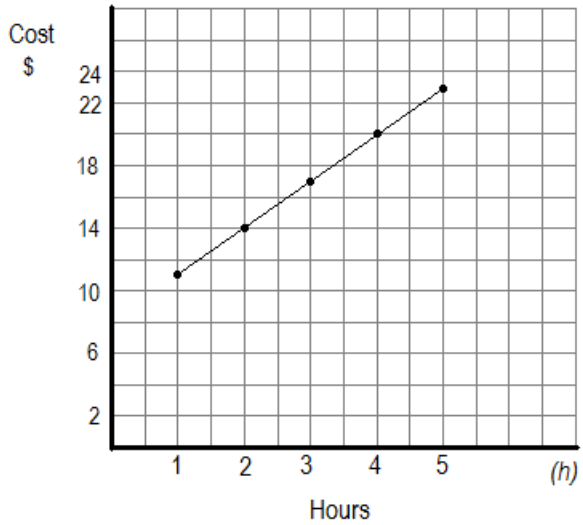
**Table**

| Number of Hours (hrs) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Cost ($) | **11** | **14** | **17** | **20** | **23** |

**Values of Expression**

8 + 3(*h*), h = 1, 2, 3, 4, 5 and 6.
8 + 3(*1*) = 11
8 + 3(*2*) = 14
8 + 3(*3*) = 17
8 + 3(*4*) = 20
8 + 3(*5*) = 23

**Plot or Graph (x, y) or (h, Cost)**



| | **11** | **14** | **17** | **20** | **23** | **…** |
|---|---|---|---|---|---|---|
| | 14 minus 11 | 17 minus 14 | 20 minus 17 | 23 minus 20 | | |
| First Difference | =+3 | =+3 | =+3 | =+3 | Constant Difference | |

An algebraic equation can be found from the table or plot.

Using the table data, the constant difference can be found. Since the first difference results with a constant difference, the equation is linear. Each consecutive hour the cost increases by three dollars. At 1hr the total costs are $11. In order to find the vale at zero, subtract $3.00 from $11.00, 11-3=8.

$$C=3h+8$$

Using the graph, extend the line to the y-axis. The value at h=0 is call the y-intercept. This value is 8.